ALGEBRAIC STRUCTURES ON GRAPH ASSOCIAHEDRA

MARÍA O. RONCO

ABSTRACT. (joint work with S. Forcey) In [1], M. Carr and S. Devadoss associated a finite partially ordered set to any simple graph Γ with a finite number of nodes. The geometric realization of this poset is a convex polytope $K\Gamma$, called the graph-associahedron. Their construction include many well-known families of polytopes, liked permutahedra, associahedra, cyclohedra and the standard simplexes.

We introduce a substitution operation on Carr and Devadoss tubings, which allows us to describe graph associahedra as a free object on the set of all connected simple graphs, for a type of colored operad generated by pairs (Γ, t), where Γ is a connected graph and t is a connected subgraph of Γ.

The linear graph is a graph L_n, with set of nodes $\{1, \ldots, n\}$ and edges $\{i, i+1\}$ for $1 \leq i \leq n-1$. A tubing of L_n is just a planar tree with $n+1$ leaves.

Substitution in this case gives the replacement of a vertex of a tree by another tree with the same number of inputs. This operation defines a monad in the category of endofunctors of graded vector spaces, which gives rise to the definition of non-symmetric operad. In general, substitution on the faces of graph associahedra induces an endofunctor of the category of graded vector spaces, but composition is not always defined. However substitution on general graph associahedra may be understood in the context of M. Batanin and M. Markl's operadic categories.

We also describe a partial order on the faces graph-associahedra, different from the one given by Carr and Devadoss, which allows us to construct a standard triangulation of graph associahedra, following [2].

REFERENCES

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[2] J.-L. Loday, *Parking functions and triangulation of the associahedron*, Proceedings of the Street's fest 2006, Contemp. Math. AMS 431 (2007), 327–340.

(Maria Ronco) INSTITUTO DE MATEMÁTICAS, UNIVERSIDAD DE TALCA, CHILE
Email address: mariaroncol@inst-mat.utalca.cl